

# Patch-Based Video Denoising with Optimal Flow Estimation

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*Abstract—A novel image sequence denoising algorithm is presented. The proposed approach takes advantage of the self similarity and redundancy of adjacent frames. The algorithm is inspired by fusion algorithms, and as the number of frames increases, it tends to a pure temporal average. The use of motion compensation by regularized optical flow methods permits robust patch comparison in a spatiotemporal volume. The use of principal component analysis ensures the correct preservation of fine texture and details. An extensive comparison with the state-of-the-art methods illustrates the superior performance of the proposed approach, with improved texture and detail reconstruction.*

*Index Terms—Video denoising, patch processing, optical flow, non-local means, motion compensation.*

## I. INTRODUCTION

Image sequence denoising is an important and open problem, less treated than the single image case. An additive white noise model is generally assumed. This simplistic model is valid for a large set of image sequences, and might be a good starting point for those a case in which is not valid. Let us denote the image sequence by  $I(x, y, t)$ , with  $(x, y)$  the spatial coordinates and  $t$  the temporal component, then this model assumes that

$$I(x, y, t) = I_0(x, y, t) + n(x, y, t) \quad (1)$$

where  $I_0$  is the true image sequence and  $n(x, y, t)$  the noise i.i.d. realizations of a Gaussian variable of zero mean and standard deviation  $\sigma$ . Since most image sequence denoising techniques rely on still image denoising algorithms, we briefly review the literature for this case. Techniques for noise removal in digital images comprise transform thresholding, local averaging, patch based methods and variational techniques.

The sliding window DCT and wavelet thresholding [18] are the main examples of thresholding methods. These methods decompose the original data in a predefined basis and cancel coefficients under a certain threshold related to noise statistics. Anisotropic filtering [2] and bilateral filter or neighbourhood filtering aim at averaging close pixels belonging to the same object, thus reducing the noise amplitude and preserving the main object boundaries. NL-means [10] introduced patch based methods into image

denoising. The algorithm groups similar patches all over the image and averages them in order to reduce noise. The method is able to preserve texture and fine details additionally to the main boundaries of the image. For a complete review of image denoising algorithms we refer the reader to [9]. Originally, the proposed denoising methods clearly fitted in one of the categories already mentioned. Nowadays, state-of-the-art methods actually combine two or three of these techniques. BM3D [15] combined patch based grouping and thresholding methods, using a 3D DCT transform. In [19] the authors introduced sparse representation in a redundant dictionary for denoising purposes.

In [43] the authors explored the use of multi-scale combined with this dictionary learning techniques. Several methods appeared combining the grouping of similar patches and the learning of an adapted basis via PCA or SVD decomposition. State-of-the-art results are obtained using Gaussian models for the group of similar patches [26] or adapting the shape of the patch before learning a PCA model [16]. Also the authors introduced a new taxonomy based on image representations for a better understanding of these state-of-the-art image denoising techniques. Local average methods, as the bilateral filter, or patch based methods as NL-means [10] or BM3D [16] and NLBayes [26] can be easily adapted to video just by extending the neighboring area to the adjacent frames.

The performance of local average methods is improved by introducing motion compensation. These compensated filters estimate explicitly the motion of the sequence and compensate the neighborhoods where similar pixels are searched for. The compensation of this search zone improves the search strategy of neighborhood filters since the single pixel color comparison is not robust enough. It was shown in [11], that a compensation of the image sequence is not necessary if a patch based algorithm is used for denoising.

The selection of the most similar patches across adjacent frames actually adapts to motion and selects similar patches wherever in the sequence. Now, the method we propose in this paper as well as VBM4D make use of motion estimation in order to define a 3D spatio-temporal patch comparison. In this sense motion estimation actually

improves denoising algorithms, which was not the case for the methods used when [11] was published.

Boulanger *et al.* [6] extended NL-means to video by growing adaptively the spatio-temporal neighborhood. Xu *et al.* The authors make use of an external database for still image denoising. The most similar images are retrieved from an external database and used for patch-based denoising which is combined with an internal denoising stage inspired by BM3D. Mairal *et al.* learnt multi-scale sparse representations for video restoration.

VBM4D, the state-of-the-art for white noise removal in image sequences, exploits the mutual similarity between 3D spatio-temporal volumes constructed by tracking blocks along trajectories defined by the motion vectors. Mutually similar volumes are grouped together by stacking them along an additional fourth dimension, thus producing a 4D structure. Collaborative filtering is realized by transforming each group through a decorrelating 4D separable transform and then by shrinkage and inverse transformation. Motion vectors are computed by block matching. Liu and Freeman [27] also use motion vectors and group patches across adjacent frames but in a different manner.

Instead of comparing patches to the reference patch, these are compared in each frame with the compensated patch of the reference one. NL-means is applied to this group of collected patches. The proposed algorithm adaptively computes the noise model of the sequence, which is an important issue for real applications. Despite the advances in video denoising, the improvement is limited compared to the one achieved in single image denoising. Noise reduction is increased by taking into account adjacent frames, but texture and detail preservation is not drastically improved. This improvement of details and texture, is usually referred in the literature as image fusion [22], [25]. Image fusion is not directly of interest in the removal of noise but in a more general restoration of the image, that is, deblurring, increase of detail or even of resolution. The key of these approaches is the use of a global registration, more robust to noise, blur and color or compression artifacts and, additionally, providing sub pixel accuracy. These global registration techniques usually rely on feature matching, for example SIFT [29], and on a parametric registration, either using an affinity or an homography.

The viewfinder alignment [1] performs such a registration by an affine function, with the important characteristic of being extremely fast. The general approach is the use of a homography [12], [20], [22], [25]. It must be noted that a homography is valid only for planar scenes or if the optical center is not modified. We propose a new algorithm making use of motion estimation algorithms and patch based methods for denoising. Our method is inspired by image fusion algorithms in the sense that it tends to a

fusion algorithm as the temporal sampling of the sequence gets dense and the motion estimation or global registration is able to perfectly register the frames and no occlusions are present. As this is an ideal scenario, our algorithm compensates the failure of these requirements by introducing patch comparison and denoising in an adapted PCA based transform.

Unlike VBM4D the motion estimation used by our algorithm relies on the optical flow constraint (OFC), that is, we suppose that the color of each pixel remains constant along its trajectory through the sequence. The optical flow is used to warp adjacent frames and not only for compensating neighborhoods. Thus, the sub pixel accuracy improves the patch comparison and averaging. Results in motion estimation are far from being totally satisfactory, there are many unsolved issues as occlusions, non translational motions, non color constancy, etc. Despite these limitations, we will show that OFC algorithms are a useful tool for denoising.

We shall proceed as follows. Section II reviews patch based methods and their main techniques. Motion estimation algorithms based on variational formulations of the OFC are presented in Section III. Section IV details the proposed algorithm and Section V discusses the choices made in its design. The experiments in Section VI compare the results of the proposed approach with three state-of-the-art methods (ND-SAFIR, VBM4D and VBM3D, the adaptation to video of the popular BM3D algorithm). Finally, in the last section some conclusions are drawn.

## **II. PATCH BASED IMAGE DENOISING**

The performance of patch-based grouping and averaging methods (introduced by NL-means [10]) has been drastically improved by the combination with transform thresholding as originally proposed in BM3D [16]. In the same paper, two additional techniques were introduced for patch based algorithms, which increase the noise reduction and the preservation of details. These techniques were already used by sliding DCT classical algorithms, but not for the newly introduced patch based ones.

First, the whole patch is denoised and not only the central pixel, which permits an increase in the noise reduction by taking the average of all estimates per pixel (aggregation). Second, the denoised image is used as guide ("oracle") for a second iteration. The similarity between two patches is computed in the first denoised image, and the transformed coefficients are used to drive the thresholding in the second iteration. The use of an adapted basis learnt from the group of similar patches is also a popular technique. In two stage image denoising, this strategy is applied twice in order to improve the noise reduction. While this strategy was shown to be less effective than the oracle iteration, the paper was the first introducing the use of PCA. Similar approaches have been proposed using the singular value decomposition (SVD) [33], which is equivalent to the

use of PCA. NL-Bayes [26] uses a Bayesian estimation algorithm: a Gaussian model is learnt for the group of similar patches and optimal denoising performed.

Finally, BM3D-SAPCA [16] combines the use of PCA with non square patches adapted at each pixel to the image itself. Patch based algorithms have also been adapted to other kinds of noise than additive white noise. For example, Deledalle et al. [17] adapted the patch distance for grouping and the estimation to any kind of noise, including the Poisson

model. Instead of modifying the algorithm formulation, a stabilization transform can be applied, permitting the use of the original denoising algorithms. For color images, a color decorrelating transform is applied before denoising. The YUV or YCrCb spaces compute a gray component  $Y$  as the average of RGB components, while the chromatic components encode the difference of the red and blue channels with the  $Y$  component. The  $Y$  component containing the geometry of the image is actually less noisy than each of the RGB components, since it is computed as the average of them. Patch comparison is performed in the  $Y$  component, while each component is denoised independently.

The smoothness of  $U$  and  $V$  permits a larger noise reduction than that obtained in the RGB space. This strategy was used by sliding DCT methods and introduced into patch based algorithms by BM3D.

This choice for denoising color images has additional reasons when dealing with Bayesian or PCA based algorithms. These algorithms need the computation of an adapted model for each patch group. In order to learn a robust model, the number of patches in the group must be larger than the dimension of the patch. Considering color patches with three times more pixels, would need of a larger group of similar patches, which is not always available with finite resolution images.

### III. OPTICAL FLOW ESTIMATION AND OCCLUSION DETECTION

Optical flow constraint based methods suppose that each pixel has the same color during the whole trajectory, or at least at adjacent frames. That is, they suppose

$$I(x, y, t) = I(x + u(x, y, t), y + v(x, y, t), t + \Delta t),$$

that where  $u$  and  $v$  are the displacement vectors at time  $t$  and pixel  $(x, y)$ . This equation alone is unable to determine the flow. The uncertainty is solved by adding a spatial or spatiotemporal regularization term. The optical flow constraint can be linearized into the well known equation,  $I_x u + I_y v + I_t = 0$ , and methods differ on how this constraint and the regularization term are imposed. The classical Horn

and Schunk [23] method used  $\psi(s) = s^2$  and  $\varphi(\nabla u, \nabla v) = |\nabla u|^2 + |\nabla v|^2$ .

It is well known that the square function excessively regularizes the discontinuities of the flow and is not robust to outliers and occlusions. Robust functions and anisotropic regularization replaced this classical approach, see for instance Alvarez *et al.* [3]. Brox et al. [7] introduced a different linearization of the OFC and a warping strategy in order to minimize the functional with  $\psi$  and  $\varphi$  functions robust to occlusions and outliers. This approach permits the introduction of additional constraints [8] on the displacement of several points. These displacements might be obtained by key point matching. The inclusion of these constraints improves the ability for capturing long range motions. Zach et al. [47] introduced total variation minimization into the flow computation. The total variation term is minimized via the Chambolle dual algorithm [13].

One of the major drawbacks of these approaches is the failure of the color constancy hypothesis, for which a constancy of the gradient [7] or the Laplacian might be additionally imposed. Recently, Wedel et al. proposed a method to decompose the sequence into a cartoon and a texture part and use only the texture part for estimating the flow. Occlusion detection can be directly taken into account by modifying the functional.

This additional term needs the setting of a new parameter amounting to fixing the percentage of the image being occluded, which is unknown a priori. Since the occluded points mostly coincide with points of negative divergence, Ballester et al. [5] introduced an additional term with the divergence of the flow as an occlusion indicator. These methods generally provide an integer precision displacement unless the image is previously zoomed in, which is time consuming. As the proposed denoising method actually resamples the sequence a real precision displacement is needed. For such reason, the proposed denoising algorithm makes use of optical flow based algorithms. We will use the total variation approach introduced by, which does not involve occlusion detection in the functional. Occlusions will be detected a posteriori following the same approach of Sand and Teller.

### IV. COMPLETE ALGORITHM DESCRIPTION

In this section we describe the complete algorithm for denoising a frame  $I_k$  from a sequence  $\{I_1, I_2, \dots, I_N\}$  (see Algorithm 1). The same procedure is applied sequentially to all the frames of the sequence.

#### Algorithm 1

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Algorithm 1 SPTWO Denoising Algorithm
Input : video sequence  $I_k, k = 1, \dots, N, I_k: \Omega \rightarrow \mathbb{R}$ .
Parameters:  $K$ = minimum number of extended patches used for denoising,  $r$ = radius of spatial neighborhood,  $t$ =radius of
temporal neighborhood ( $M$ =number of frames in neighborhood= $2t + 1$ )
Output : denoised sequence  $\hat{I}_k, k = 1, \dots, N$ 
for  $k = 1, \dots, N$  do
    //Create aligned sequence and detect occluded pixels
     $N_k^t = \{j \in 1, \dots, N/|j-k| \leq t\}$  //Temporal neighborhood of frame  $k$ 
    for  $j \in N_k^t$  do
        if  $j \neq k$  then
             $w_{kj}(x) = \text{Optical\_Flow}(I_k, I_j)$  //Optical flow from  $I_k$  to  $I_j$ 
        else
             $w_{kk}(x) = 0$ 
         $I_j^w(x) = I_j(w_{kj}(x))$  //Warp frame  $I_j$  using the computed flow
        //Build mask using Definition (2), with  $I_0 = I_k, I_1 = I_j$ .
        //Pixels with  $w(x) < 0.5$  are labeled as "occluded".
         $M_j(x) = \text{Occlusion\_Mask}(I_k(x), I_j(x))$ 
        Remove frame  $j$  from  $N_k^t$  if  $M_j(x) = \text{"occluded"}$ 
    //Denoise frame  $k$ 
    for each pixel  $x \in \Omega$  do
         $\mathcal{P}_x = \emptyset$  //Extended patch at  $x$ : set of patches associated to pixel  $x$ 
        for  $j \in N_k^t$  do
             $P_j(x) = \text{Patch centered at } x \text{ in frame } I_j$ 
             $\mathcal{P}_x = \mathcal{P}_x \cup P_j(x)$ 
         $N_{x=(x_1, x_2)}^r = \{y = (y_1, y_2) / |x_1 - y_1| \leq r \text{ and } |x_2 - y_2| \leq r\}$  //Spatial neighborhood of  $x$ 
         $\mathcal{D}_x = \emptyset$  //Set of differences between sets of patches
        for each pixel  $y \in N_x^r$  do
             $\mathcal{P}_y = \emptyset$  //Extended patch at  $y$ : set of patches associated to pixel  $y$ 
            for  $j \in N_k^t$  do
                 $P_j(y) = \text{Patch centered at } y \text{ in frame } I_j$ 
                 $\mathcal{P}_y = \mathcal{P}_y \cup P_j(y)$ 
            //Compute difference between sets of patches
             $d_{xy} = d_{SD}(\mathcal{P}_x, \mathcal{P}_y) = \sum_{j \in N_k^t} \|P_j(x) - P_j(y)\|^2$ 
             $\mathcal{D}_x = \mathcal{D}_x \cup d_{xy}$ 
         $\mathcal{D}_x = \text{Sort}(\mathcal{D}_x)$  //Sort differences in increasing order
         $\mathcal{S} = \emptyset$  //Set of patches used for denoising
         $\hat{\mathcal{P}} = \emptyset$  //Set of denoised patches
        //Read sorted set of differences
        for  $d_{xy} \in \mathcal{D}_x$  and  $\text{card}(\mathcal{S}) < K \cdot M$  do
            for  $j \in N_k^t$  do
                if  $P_j(y)$  does not contain any "occluded" pixel then
                     $\mathcal{S} = \mathcal{S} \cup P_j(y)$ 
             $\hat{P}_k(x) = \text{PCADenoising}(\mathcal{S})$  //Denoised patch
             $\hat{\mathcal{P}} = \hat{\mathcal{P}} \cup \hat{P}_k(x)$ 
         $\hat{I}_k = \text{AggregatePatches}(\hat{\mathcal{P}})$ 

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### A. Motion Compensation

First, the optical flow between  $I_k$  and adjacent frames in a temporal neighborhood is computed and used for warping these frames onto  $I_k$ . If registration was accurate and the sequence free of occlusions, a temporal average in this aligned data would be optimal, even if the noise reduction would slowly decrease as  $1/M$ , where  $M$  is the number of adjacent frames involved in the process. Generally, this will not be the case, inaccuracies and errors in the computed flow and the presence of occlusions make this temporal average likely to blur the sequence and have artifacts near occlusions. The proposed approach tends to solve these limitations. Occlusions are detected depending on the divergence of the computed flow: negative divergence values indicate occlusions. Additionally, the color difference is checked after flow compensation. A large difference indicates occlusion, or at least failure of the color constancy assumption.

### B. Choice of Similar Patches

Let  $\{I_{w_{k-t}}, \dots, I_{w_{k+t}}\}$  be the set of adjacent frames to  $I_k$  after warping with the computed optical flow, and let  $M_j$  be the occlusion mask between frames  $I_k$  and  $I_{w_j}$ ,  $j \in \{k-t, \dots, k+t\}$ . The algorithm uses a 3D volumetric

approach to search for similar patches, while still 2D image patches are used for denoising.

For each  $n \times n$  patch  $P$  of the reference frame  $I_k$ , we consider the patch  $P$  referring to its extension to the temporal dimension, having  $M$  times more pixels than the original one (assuming  $M$  patches in the temporal neighborhood,  $M = 2t + 1$ ),  $P = (P_{k-t}, \dots, P_{k+t})$ . The algorithm looks for the  $K$  extended patches closest to  $P$ . As each extended patch contains  $M$  2D image patches, the group contains  $K \cdot M$  selected 2D patches of size  $n \times n$ . The Principal Component Analysis (PCA) of these patches is computed and their denoised counterparts are obtained by thresholding of the coefficients.

### C. Denoising of Similar Patches

The PCA analysis of this set of  $M \cdot K$  patches looks for the basis of  $R_{n2}$  better explaining its structure in the sense that most of the information describing all patches is concentrated in a few vectors of the basis. The amount of information that each vector of the basis conveys is coded in the  $n_2$  principal values. That is, by keeping only the coefficients associated to the most important vectors (the ones with highest corresponding principal value) we keep the maximum of information while discarding coefficients related to less important vectors that we remove noise. The decision of canceling a coefficient of a certain patch is not taken depending on its magnitude, but the magnitude of the associated principal value. A more robust thresholding is obtained by comparing the principal values to the noise standard deviation and canceling or maintaining the coefficients of all the patches associated to a certain principal direction.

### D. Second "Oracle" Iteration

A second iteration of the algorithm is performed using the "oracle" strategy. Once the whole sequence has been restored, we re-apply the algorithm on the initial noisy sequence, but motion estimation and patch selection are performed on the result of the first iteration

### E. Flat Parts Processing

For large values of noise standard deviation some residual noise might be left in flat zones. This is a common issue to image denoising methods using an adaptive basis [26]. The reason is that in flat zones, the selection of patches is taking into account only noise because of the lack of geometry in these parts, which includes a bias in the selection process. In addition, the PCA analysis with pure noise patches and a reduced number of samples might generate principal values larger than expected. We apply the same solution to this issue as proposed in [26]. Whenever a set of patches is detected to be flat up to the noise oscillations, a simple average of all values in the patches is taken instead of computing a PCA model.

### F. Color Image Denoising

Color images are denoised directly without the use of any color decorrelating transform. Each color patch is considered as a vector with three times more components than in the single channel case. The use of several frames makes the number of patches available much larger, relaxing the conditions on the length of the patch vector for learning an adapted model. This permits the use of PCA with color patches. That is, the color decorrelation of PCA is adapted for each group of patches, thus increasing the effectiveness of the model. The final color algorithm is applied to vectorial patches, following the steps described in Algorithm 1. The optical flow is in this case computed on the gray version of the color images. Each channel is resampled with the same flow, and the same mask of occlusions is used for all the channels.

## V. DISCUSSION

### A. Impact of Motion Estimation and Resampling

We cannot use our algorithm directly with block matching (BM) motion estimation. The BM gives an integer (or pixel) precision flow, while our algorithm requires subpixel precision in order to achieve accurate warping of the images. The use of non-integer pixel locations implies a resampling of the image, i.e. an interpolation of its values. We can indeed apply the algorithm without the resampling step, in which case both block matching and optical flow are valid. If the sequence is not resampled, the spatio-temporal patch distance is computed compensating the 2D patches taking into account the flow. In this case, the flow indicates which are the coordinates of the patch in the corresponding frame (integer precision). The rest of the denoising algorithm is as described in the previous section. Here the figure compares the application of the non-resampling strategy with optical flow and block matching algorithms and the resampling strategy with optical flow (our proposed algorithm).



**Fig 1. Discussion on motion estimation and resampling with noise standard deviation 20. From left to right: crop of the noisy central image, denoised by the algorithm with optical flow estimation and non resampling, denoised by the algorithm with block matching estimation and non resampling and proposed algorithm. The RMSE is respectively 4.92, 5.05 and 4.46.**

### B. 3D vs 2D Patch Denoising

The correct selection of similar patches is crucial for both image and video denoising. When processing single images we are forced to increase the size of the patch when dealing with severe noise, which at the same time limits the number of similar patches found. The use of 3D patches allows comparison with many more pixels thus being more robust without the need of increasing the 2D support. When using a PCA decomposition there is always a tradeoff between the dimension of the vectors used (number of pixels of the patches) and the number of samples (number of similar patches). In order to correctly estimate the PCA we need the number of samples to be larger than the dimension of the vectors. This is the main limitation for using 3D patches in the filtering. If we were to use 3D patches of dimension  $n \times n \times M$  for filtering, we would need more than  $Mn^2$  similar 3D patches, but we only have  $K$  of them ( $K$  denoting the number of similar 2D windows in the reference image, with  $K$  usually smaller than  $Mn^2$ ). Whereas by using 2D patches we have available  $M \cdot K$  samples of dimension  $n \times n$ . Another drawback of using 3D patches is the presence of occlusions, since we remove 2D patches which have occlusion we couldn't use the 3D patches containing them for denoising.

## RESULTS:

## VII. CONCLUSION FUTURE WORK

We presented a novel denoising algorithm combining motion estimation and patch based denoising algorithms. Motion compensation permits the use of spatio-temporal patches for a more robust comparison while the use of PCA for patch denoising preserves texture and details. The comparison with state-of-the-art algorithms illustrates the gain on performance of the proposed approach. The next step is the extension of the current algorithm to other type of noise models, including correlated noise but also compression artifacts of video standard compression algorithms.

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