

Optimal Power flow Model with Storage and Renewable in IEEE 14 Bus test systems

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Abstract: Renewable energy sources and energy storage systems present specific challenges to the traditional optimal power flow (OPF) paradigm. First, storage devices require the OPF to model charge/discharge dynamics and the supply of generated power at a later time. Second, renewable energy sources necessitate that the OPF solution accounts for the control of conventional power generators in response to errors of renewable power forecast, which are significantly larger than the traditional load forecast errors. This paper presents a sparse formulation and solution for the affinely adjustable robust counterpart (AARC) of the multi-period OPF problem. The AARC aims at operating a storage portfolio via receding horizon control; it computes the optimal base-point conventional generation and storage schedule for the forecasted load and renewable generation, together with the constrained participation factors that dictate how conventional generation and storage will adjust to maintain feasible operation whenever the renewables deviate from their forecast. The approach is demonstrated on standard IEEE networks dispatched over a 24-h horizon with interval forecasted wind power, and the feasibility of operation under interval uncertainty is validated via Monte Carlo analysis. The computational performance of the proposed approach is compared with a conventional implementation of the AARC that employs successive constraint enforcement.

Index Terms—Energy storage, integer linear programming, optimal power flow, optimization methods.

I. INTRODUCTION

THE single-period optimal power flow (OPF) models the network constraints in the dispatching solution and plays an important role in the generation control function of an energy management system [1], [2]. The inter-temporal constraints comprise the generation ramp-rate bounds together with the generation minimum-up/minimum-down

time limitations. The common practice is to include the inter-temporal constraints in the unit commitment problem that schedules the hours during which the generating units must be ready to run [3], with the generation dispatch subsequently computed by solving a static OPF problem for each period on its own. However, the advent of energy storage systems (ESS) such as batteries, flywheels, and compressed air makes generation dispatching incompatible with the current single-period OPF practice; this is due to the operation of storage being strongly coupled over the time periods [4]. A viable solution in this case is receding horizon control (RHC), in which a multi-period OPF is solved over a finite horizon, the first step of the solution is executed, and the process is repeated for the next time step with the most recently available information [5]. In fact, multi-period OPF has been already employed for solving look-ahead dispatch problems for generation with ramp-rate constraints [6]. ESS are normally deployed in networks having renewable energy sources (RES), where the ESS capacity to transfer power over time proves to be invaluable. The variability of generation from RES requires specific modeling to ensure network balance at each time step of the multi-period OPF, and maintaining secure operation remains an active research problem. The OPF with storage was formulated as a finite-horizon optimal control problem in [7], where an explicit optimal solution for the charging pattern is found in the case of a single generator connected to a single load by a line of infinite capacity. For the general case with several generators and loads, the formulation is extended to a multi-period OPF that can employ either a DC [7] or an AC [8], [9] network model. With a DC network model and convex cost curves, the problem can be solved using convex optimization software. The AC network model leads to a non-convex optimization problem, where guarantees on the performance of the solver and the quality of the solution generally cannot be given. However, recent findings in [10] show that the single-period OPF with quadratic cost functions admits a

convex semi definite programming (SDP) relaxation which will give the correct answer in many practical instances; this relaxation has been extended in [11] and [12] to the case of multi-period OPF with ESS, though computational performance on large systems remains a bottleneck. Ref. [13] studied the relation between locational marginal prices and storage dynamics in the context of the multi-period SDP-OPF, and gave conditions under which simultaneous charging and discharging will not occur. In other multi-period OPF applications, simultaneous charging and discharging was avoided by defining a predetermined cycle of charge/discharge per day [14]; further flexibility in scheduling the storage operation comes at the expense of solving a mixed-integer nonlinear programming formulation [15], [16], which is an NP-hard problem. The formulations [7]–[16] can seamlessly account for renewable generation as a deterministic quantity, essentially by treating it as negative load. The deterministic assumption implies an accurate short-term forecast over the dispatching horizon; while this is commonly accepted for loads, it is far less justified for RES [17]. To extenuate the intermittency effects of wind power generation, [18] proposed scheduling fast reserves via a risk-mitigated OPF that uses chance constraints with a Gaussian model for the prediction errors. In current operational procedures, optimal power flow dispatching is employed to compute the base-point generation, and participation factors are used to adjust the conventional generation as load varies between base-point optimizations [19], [20]; the same principle applies for handling RES variability [21]. The calculation of participation factors that guarantee security before the next optimization has been proposed in [20] and further improved in [2]. Typically computed participation factors with base-points from a deterministic OPF solution have proved to work well under load fluctuations [1], [2], [19], [20], but can give rise to significant line overloads under the larger wind power variations. Reference [22] proposed computing the participation factors for Gaussian wind power fluctuations by solving a conic reformulation of the chance constrained problem. The practical application of stochastic optimization requires accurate statistical models that may not be easily available [5], and this has motivated the study of problems related to data errors in chance-constrained approaches [22]. From an implementation point of view, the methods in [1], [2], [20], and [22] for computing the generation participation make use of a single-period OPF formulation and are therefore not directly applicable for handling storage charge/discharge dynamics. This paper

proposes an affinely adjustable robust counterpart (AARC) of the multi-period OPF problem; with convex piecewise linear cost curves and a DC network model, the AARC is a mixed-integer linear program which allows scheduling conventional generation/ESS and computing constrained participation factors that remain valid as the renewable generation varies over its uncertainty interval. The binary variables in the AARC preclude simultaneous charging/discharging of ESS, and the participation factors command the conventional generation and ESS adjustments for maintaining a feasible multi-period OPF solution as the RES output takes any value from a prescribed uncertainty set. In fact, the literature on short-term wind prediction presents confidence intervals of the predicted values [17] and [23], [24] recently propose a methodology for direct interval forecasting; the confidence or forecast intervals are synonymous with the intervals of uncertainty in the context of robust optimization [25] and serve as motivation for this work. Additionally, the AARC conforms with the automatic generation control functionality under the current power engineering practice [19]. The proposed AARC formulation also maintains sparsity of the multi-period OPF model; this is shown to be advantageous for computational performance and results in significantly reduced storage requirements. Unlike [26], it employs a compact robust reformulation of the generator dispatch and ramp-rate constraints, and accounts for storage devices with participation factors that dictate the charge/discharge power adjustment when the uncertainty is revealed. The rest of this paper is organized as follows. Section II reviews recent developments in two-stage robust optimization and Section III presents the multi-period OPF formulation with RES and ESS. Section IV introduces the proposed sparse AARC, with the full-set power flow equations and a compact representation of robust generator and storage limitations; the compact reformulations in Section IV-A and IV-B do not make use of the general AARC format in [25]. Section V presents numerical results that are validated via Monte Carlo simulation; additionally, the computational performance is contrasted with the conventional AARC [25], which requires the use of the reduced-set or non-sparse power flow formulation [26]. The paper is concluded in Section VI. Practical use of software for multi-stage real-time dispatch is already adopted by the PJM Interconnection [27], implying that the ideas for real-time dispatch reported herein can be of value for system operators.

II. ROBUST FORMULATIONS

Robust optimization has emerged as a powerful method for optimization under uncertainty. In particular, two-stage robust optimization with complete recourse has been recently proposed for solving power system scheduling and unit commitment problems [28]–[31]. The two-stage decision environment distinguishes between the here-and-now (nonadjustable) variables that must be determined before the realization of the uncertainty, and the wait-and-see (adjustable) variables that adjust themselves once the uncertainty is revealed; in this context, complete recourse implies that the second-stage variables are free to adjust within their feasible operating range so as to restore feasibility. Two-stage robust optimization with complete recourse is in most cases NP-hard [32], and exactly solving the second-stage problem with general polyhedral uncertainty sets remains computationally intensive. Despite the usefulness of the two-stage robust solution in multi-period optimization problems, the formulation inherently assumes that the uncertainty is simultaneously revealed in all intervals; the solution to multistage robust optimization problems remains a subject for research. A finely adjustable robust optimization offers a tractable implementation for two-stage robust optimization problems; this comes at the expense of restricting the second-stage variables to adjust as affine functions of the uncertainty [25], [32]. The solution of the AARC is expected to be suboptimal relative to the two-stage robust solution with complete recourse. However, research in [33] and [34] suggests that in some cases the affinely adjustable solution is indeed optimal, and experience with power system dispatch problems is encouraging [26]. Recent research on generalized decision rules [35] aims to reduce sub-optimality as compared to the robust solution via affine policies; however, its practical applicability to multistage power system optimization problems is yet to be determined

III. MULTI-PERIOD OPTIMAL POWER FLOW PROBLEM

Consider a power network n having nodes with demand ($P_{Di}(t)$), conventional generation ($P_{Gi}(t)$) and RES ($P_{Ri}(t)$) connected to them storage is defined in accordance with the generation convention i.e is the storage power is positive ($P_{si}(t) = P_{si}^d(t)$) during discharge periods and negative ($P_{si}(t) = -P_{si}^c(t)$) during charge periods. The index i runs from 1 to n with

the provision that any quantity (such as storage power and conventional/renewable generation) that is not associated with node i is set to zero

$$P_{Gi}(t) = 0, i \notin G; P_{Ri}(t) = 0, i \notin R; P_{Si}(t) = 0, i \notin S(1)$$

The multi-period OPF computes the base-point generation values and the schedule of the storage portfolio; its objective is minimizing the cost of operating conventional generation over the dispatch horizon ($t=1, \dots, T$), given the demand and the renewable power forecast [12]:

$$\sum_{t=1}^T \sum_{i \in G} \sum_{K=1}^{N_i} (C_{ik} P_{Gik}(t) + C_{io}) \Delta t(2)$$

The physical and technical constraints that govern the multiperiod OPF problem are listed below:

- Generator dispatch and ramp-rate limits

$$P_{Gi} = P_{Gi}^{min} + \sum_{k=1}^{N_i} P_{Gik}(t), i \in G, t = 1, \dots, T(3)$$

$$0 \leq P_{Gi}(t) \leq P_{Gi}^{(K)} - P_{Gi}^{(K-1)}, i \in G, K = 1, \dots, N_i, t = 1, \dots, T(4)$$

$$-RR_i^{max} \Delta t \leq P_{Gi}(t) - P_{Gi}(t-1) \leq RR_i^{max}, i \in G, t = 1, \dots, T(5)$$

- DC powerflow equations and line flow limits

$$P_{Gi}(t) + P_{si}^d(t) - P_{si}^c(t) \sum_{j=1}^n B_{ij} \theta_j(t) = P_{Di}(t) -$$

$$P_{Ri}(t), \theta_1(t) = 0, i = 1, \dots, n, t = 1, \dots, T(6)$$

$$P_{ij} = b_{ij}(\theta_i(t) - \theta_j(t)) \leq p_{ij}^{max}, j \in \Omega(i), i = 1, \dots, n, t = 1, \dots, T(7)$$

- Storage dynamics, capacity, and charge/discharge power limits

$$E_i(t) - E_i(t-1) = \left(n_c P_{si}^c(t) - \frac{1}{n_d} P_{si}^d(t) \right) \Delta t, i \in S, t = 1, \dots, T$$

$$0 \leq P_{si}^c(t) \leq P_{si}^{c(max)} \beta_i(t), 0 \leq P_{si}^d(t) \leq P_{si}^{d(max)} (1 - \beta_i(t))(8)$$

$$\beta_i(t) \in \{0, 1\}, i \in S, t = 1, \dots, T(9)$$

$$E_i^{min} \leq E_i(t) \leq E_i^{max}, i \in S, t = 1, \dots, T(10)$$

$$E_i(T) = E_{if}, i \in S(11)$$

Equation (3) expresses the conventional generation power in function of its interval components ($P_{Gik}(t)$), where each is bounded by the limits in (4); the corresponding piecewise linear generator cost curve in (2) is assumed to be

convex (Fig. 1), in which case separable programming guarantees the validity of the piecewise linear formulation [36]. Equation (5) enforces the generation ramping limits that constrain the maximum allowable change between two consecutive intervals. The power network is modeled by a DC power flow (6) in each time interval, and the branch power flow constraints (7) are expressed in terms of the nodal angles; the power flow P_{ij}^{MAX} limits are enforced at each end of the branch, which is equivalent to requiring the power flow along branch I, J to be limited between flow P_{ij}^{MAX} , $-P_{ij}^{\text{MAX}}$. The storage dynamics are given by (8); (9) sets the storage charge/discharge limits and prevents simultaneous charging and discharging, which is unrealistic for most storage technologies. Equation (10) captures the minimum and maximum capacity limits for storage; (11) is optionally used for setting the final value of the stored energy to a pre-specified value, which is commonly chosen equal to the initial value $E_1^{(0)}$. The problem given by (2)–(11) is a sparse mixed-integer linear program, where the binary variables ensure that storage operates exclusively either in the charge or discharge mode. In this respect, [13] showed that simultaneous charging and discharging will not occur at a bus if the locational marginal price (LMP) at this bus is strictly positive. Congested power networks could cause negative LMPs at some buses even if

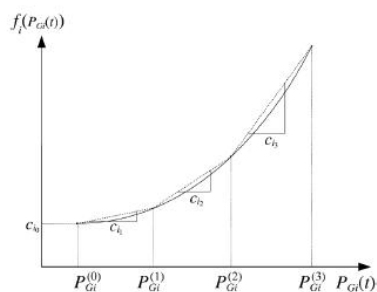


Fig. 1. Convex generation cost function.

all offer curves are positive, in which case the operational cost will be reduced if consumers draw more power from the bus with the negative LMP; this is also known to appear in the classical transportation problem, where it is named the *more-for-less-paradox* [37]. Consider an OPF solution whereas certain bus has a negative LMP; if a storage device (with a round trip efficiency less than 100%) is installed at this bus, then simultaneous charging and discharging will result in a more economical solution as it effectively increases the demand through dissipating power in the device. This is guaranteed not to happen in the above formulation due to the binary variables, where

computational experience shows that the mixed-integer linear programming (MILP) formulation is not very taxing in terms of execution time because the majority of storage is naturally scheduled in one mode [9].

IV AARC of the Multi-Period OPF Problem

The AARC of the multi-period OPF is the main computational engine behind the RHC implementation, where the AARC solution is executed for the first time step and recomputed at the next step with updated forecast estimates and confidence intervals; this aims at maintaining a non-empty feasible region when the subsequent intervals are optimized. The objective of the AARC is to minimize the base-point generation cost over the dispatch horizon:

$$\sum_{t=1}^T \sum_{i \in G} \sum_{k=1}^{N_i} (C_{ik} P_{Gik}(t) + C_{i0}) \Delta t \quad (12)$$

subject to the following constraints:

- Generator dispatch and ramp-rate limits.
- Storage dynamics, capacity, and charge/discharge power limits.
- DC power flow equations and line flow limits.
- Participation factor constraints.

The above problem is a sparse mixed-integer linear program that is solvable using available commercial software packages [38]; its solution gives the base-point generation, the storage schedule, and constrained participation factors for both conventional generation and ESS over the study horizon. Alternatively, the generation participation factors can be set to fixed values, following for instance the standard in [19]. Under some additional assumptions, the expected cost of re-dispatch can be used as an alternative objective function in terms of and; typical assumptions require the availability of the first and second moments of together with the use of quadratic cost curves [26].

A successive constraint enforcement scheme for the power flow limits (49), (50) is employed to improve the computational performance of the solution. The scheme starts by solving sub-problem that does not include any power flow limit constraints. This is followed by checking for violated power flow limit constraints, which are subsequently added to the sub problem. The iterative constraint enforcement process is repeated until all power flow limit constraints are satisfied.

V. MALAB RESULTS

The multi period OPF was programmed in MATLAB. The simulations were carried out on a

WINDOWS having a 2.7-GHz quad-core Intel Core i5 processor with 4-MB L3 cache and 8 GB of RAM. The scheduling results are reported on the IEEE 14-bus networks whose original data sets are given with the distribution files of MATPOWER and modified according to [20].

The convex quadratic generation cost curves were replaced with a three-segment linear approximation, where each segment approximates the generator quadratic cost curve over one third of its dispatch range. Each network was simulated over a period of 24 h with the time discretized into 30-min intervals [5], i.e. The half-hourly bus load profile was obtained by scaling the daily peak power at each bus (as given in the data files [21]) using the percentage values shown in Fig. 2. Storage was assumed to be installed at each bus that does not have conventional generation, with the following parameters [11], [12]: storage capacity is 32MWh and the maximum charge/discharge power limit is 8 MW. The initial and final storage energy levels were set to 16 MWh.

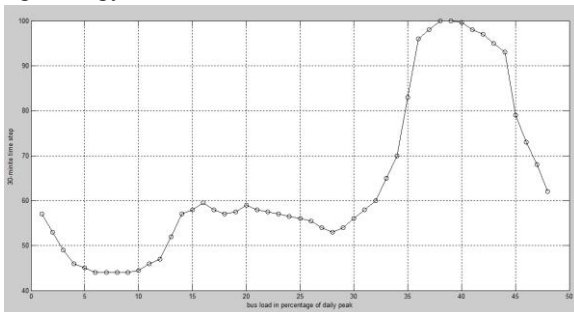


Fig. 2. Half-hourly bus load in percent of the bus load daily peak.

Table 1

Load flow of IEEE 14 bus without RES

How many	No. of	P (MW)	Q (MVar)
Buses	14		
Total Gen Capacity		772.4	-52.0 to 148.0
Generators	5		
On-line Capacity		772.4	-52.0 to 148.0
Committed Gens	5		
Generation (actual)		268.3	67.6
Loads	11		
Load		259.0	73.5
Fixed	11		
Fixed		259.0	73.5
Branches	20		
Losses ($I^2 * Z$)		9.29	39.16

Table 2

Load flow of IEEE 14 bus with RES

How many	No. of	P (MW)	Q (MVar)
Buses	14		
Total Gen Capacity		772.4	-52.0 to 148.0
Generators	5		
On-line Capacity		772.4	-52.0 to 148.0
Committed Gens	5		
Generation (actual)		268.3	67.6
Loads	11		
Load		259.0	73.5
Fixed	11		
Fixed		259.0	73.5
Branches	20		
Losses ($I^2 * Z$)		7.430	31.33

VI. CONCLUSION

This paper described a novel sparse formulation for the affinely adjustable robust counterpart of the multi-period OPF problem with RES and storage. The proposed formulation is an MILP problem that models the storage device dynamics such that simultaneous charging and discharging does not occur, and includes adjustable variables that restore solution feasibility after the uncertainty associated with the renewable power forecast is revealed. Unlike the conventional AARC formulation that employs a reduced-set power flow, the proposed AARC maintains the sparse network equations and shows significant computational advantages on larger networks with small time resolution; this is particularly important because the multi-period OPF is intended for cyclical execution in the implementation of receding horizon control. The AARC can handle the generation and storage participation factors either as optimizable variables or as fixed parameters. However, the Matlab tests demonstrate that optimizing the generation participation factors is superior to the use of the classical standard values in terms.

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